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18MAT11

# First Semester B.E. Degree Examination, July/August 2021
## Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

**Note:** ***Answer any FIVE full questions.***

**1** a. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Find the radius of curvature at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ for the curve $x^3 + y^3 = 3axy$. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

**2** a. Find the pedal equation of $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Show that for the curve $r^2 = a^2 \cos 2\theta$ the radius of curvature $\rho = \dfrac{a^2}{3r}$. **(06 Marks)**

c. Find the angle between the curves $r = a \log\theta$ and $r = \dfrac{a}{\log\theta}$. **(08 Marks)**

**3** a. Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} + \ldots\ldots$ **(06 Marks)**

b. Evaluate i) $\lim\limits_{x \to 0}\left(\dfrac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ ii) $\lim\limits_{x \to 0}(\cos x)^{\frac{1}{x^2}}$ **(07 Marks)**

c. Show that the function $xy(a - x - y)$ is maximum at $\left(\dfrac{a}{3}, \dfrac{a}{3}\right)$. Hence find maximum value if $a > 0$. **(07 Marks)**

**4** a. If $U = f(x - y, y - z, z - x)$ show that $\dfrac{\partial U}{\partial x} + \dfrac{\partial U}{\partial y} + \dfrac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If x, y, z are the angles of triangle find the maximum value of sinx siny sinz. **(07 Marks)**

c. Find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$ where $U = x^2 + y^2 + z^2$, $V = xy + yz + zx$ and $W = x + y + z$. **(07 Marks)**

**5** a. Evaluate $\displaystyle\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dx\,dy\,dz$ **(06 Marks)**

b. Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(07 Marks)**

c. Prove that $\displaystyle\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \cdot \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**



Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$. **(06 Marks)**

b. Find the volume of the solid bounded by the planes x = 0, y = 0, z = 0 x + y + z = 1. **(07 Marks)**

c. Derive the relation between Beta and Gamma function as $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(06 Marks)**

b. Find the orthogonal trajectory of $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, λ is parameter. **(07 Marks)**

c. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(07 Marks)**

8 a. Solve the L-R circuit $L\frac{dI}{dt} + RI = E$ Initially I = 0 when t = 0. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$. **(07 Marks)**

c. Solve $yp^2 + (x - y)p - x = 0$. **(07 Marks)**

9 a. Find the rank of the matrix

$$\begin{pmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{pmatrix}$$

by applying elementary row operations. **(06 Marks)**

b. Find the largest eigen value and the corresponding eigen vector for $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ with initial vector $(1\ 1\ 1)^T$ [carryout 5 iterations]. **(07 Marks)**

c. Investigate the values of λ and μ such that the system of equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + λz = μ may have i) Unique solution ii) Infinite solution iii) No solution. **(07 Marks)**

10 a. Solve the following system of equation x + y + z = 9, x – 2y + 3z = 8, 2x + y – z = 3 by Gauss elimination method. **(06 Marks)**

b. Reduce the matrix $\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ into diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss-Seidal method. 20x + y – 2z = 17, 3x + 20y-z = -18, 2x – 3y + 20z = 25 [carryout three iterations]. **(07 Marks)**

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